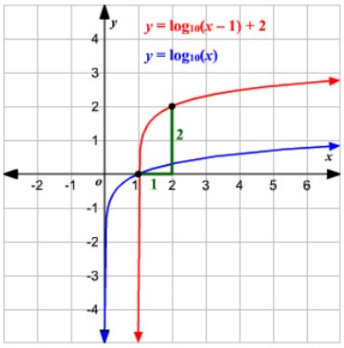
Logarithmic Functions

Objectives To graph logarithmic functions



When graphing transformations of $f(x) = \log_b x$ where $b > 1$, it helps to consider the effect of the transformations on the following features of the graph of $f(x)$: the vertical asymptote, $x = 0$, and two reference points, $(1, 0)$ and $(b, 1)$. The table lists these features as well as the corresponding features of the graph of $g(x) = a \log_b (x - h) + k$.

Function	$f(x) = \log_b x$	$g(x) = a \log_b (x - h) + k$
Asymptote	$x = 0$	$x = h$
Reference point	$(1, 0)$	$(1 + h, k)$
Reference point	$(b, 1)$	$(b + h, a + k)$

Logarithmic Functions

take note

Concept Summary Families of Logarithmic Functions

Parent functions:	$y = \log_b x, b > 0, b \neq 1$
Stretch ($\lvert a \rvert > 1$) Compression (Shrink) ($0 < \lvert a \rvert < 1$) Reflection ($a < 0$) in x-axis	$y = a \log_b x$
Translations (horizontal by h; vertical by k)	$y = \log_b (x - h) + k$
All transformations together	$y = a \log_b (x - h) + k$

Logarithmic Functions

The parent function $y = log_{10}(x)$

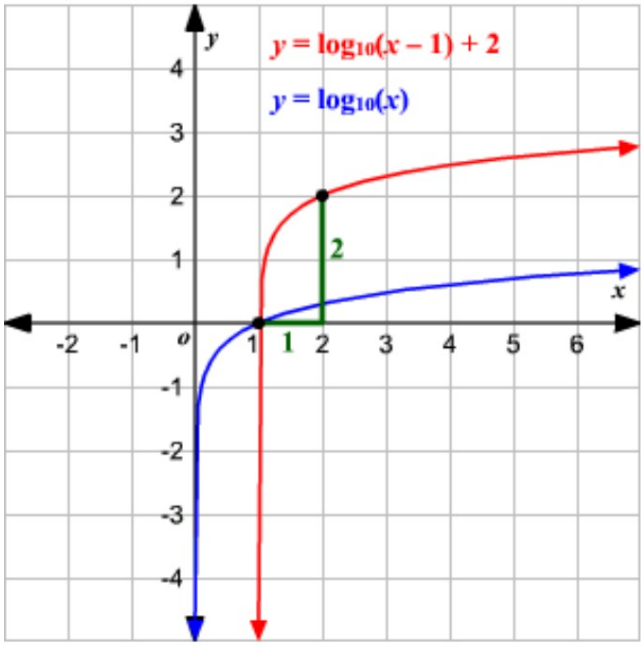
Domain: $\{x | x > 0\}$

Range: $\{y | -\infty < y < +\infty\}$

The asymptote : **y–axis**

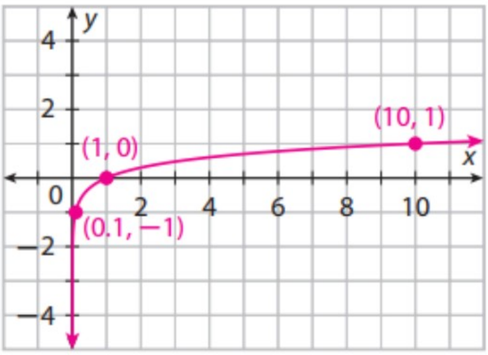
The reference points : ***(1, 0)***

(10, 1)



Logarithmic Functions

Common Log

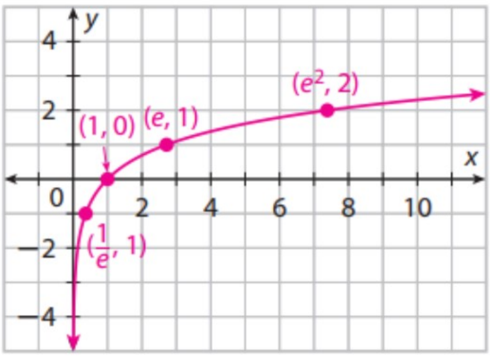


$f(x) = \log x$

Log to the base 10 of x

$f(x) = \log_{10} x$

Natural Log

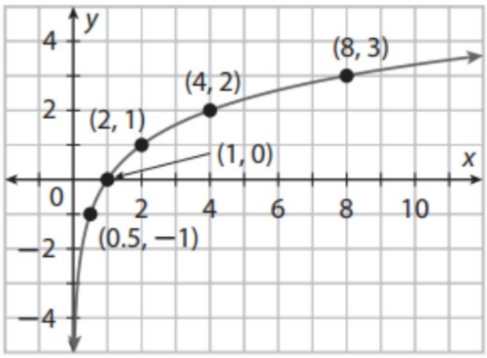


$f(x) = \ln x$

Log to the base e of x

$f(x) = \log_e x$

Log base 2 of x



$f(x) = \log_2 x$

Log to the base 2 of x

$f(x) = \log_2 x$

Logarithmic Functions

Graphing Combined Transformations of $f(x) = \log_b x$ Where $b > 1$

When graphing transformations of $f(x) = \log_b x$ where $b > 1$, it helps to consider the effect of the transformations on the following features of the graph of $f(x)$: the vertical asymptote, $x = 0$, and two reference points, $(1, 0)$ and $(b, 1)$. The table lists these features as well as the corresponding features of the graph of $g(x) = a \log_b (x - h) + k$.

Function	$f(x) = \log_b x$	$g(x) = a \log_b (x - h) + k$
Asymptote	$x = 0$	$x = h$
Reference point	$(1, 0)$	$(1 + h, k)$
Reference point	$(b, 1)$	$(b + h, a + k)$

Logarithmic Functions

Graphing Combined Transformations

(A) $g(x) = -2 \log_2 (x - 1) - 2$

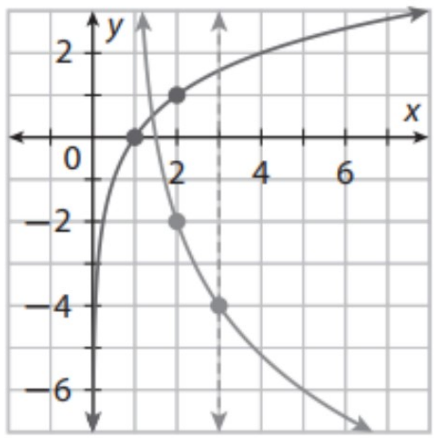
The transformations of the graph of $f(x) = \log_2 x$ that produce the graph of $g(x)$ are

- a vertical stretch by a factor of 2
- a reflection across the x-axis
- a translation of 1 unit to the right and 2 units down

Function	$f(x) = \log_2 x$	$g(x) = -2 \log_2 (x - 1) - 2$
Asymptote	$x = 0$	$x = 1$
Reference point	$(1, 0)$	$(1 + 1, -2(0) - 2) = (2, -2)$
Reference point	$(2, 1)$	$(2 + 1, -2(1) - 2) = (3, -4)$

Domain: $\{x \mid x > 1\}$

Range: $\{y \mid \infty < y < +\infty\}$



Logarithmic Functions

Identify the transformations of the graph of $f(x) = \log_b x$ that produce the graph of the given function $g(x)$.

$$g(x) = 2\log(x + 2) + 4$$

The transformations of the graph of $f(x) = \log_2 x$ that produce the graph of $g(x)$ are

- a vertical stretch by a factor of 2
- a translation of 2 units to the left and 4 units up

Function	$f(x) = \log x$	$g(x) = 2\log(x + 2) + 4$
Asymptote	$x = 0$	$x = -2$
Reference point	$(1, 0)$	$(1 - 2, 2(0) + 4) = (-1, 4)$
Reference point	$(10, 1)$	$(10 - 2, 2(1) + 4) = (8, 6)$

Logarithmic Functions

Identify the transformations of the graph of $f(x) = \log_b x$ that produce the graph of the given function $g(x)$.

$$g(x) = -2\log_2(x-1) - 2$$

The transformations of the graph of $f(x) = \log_2 x$ that produce the graph of $g(x)$ are

- a vertical stretch by a factor of 2
- a reflection across the x-axis
- a translation of 1 unit to the right and 2 units down

Function	$f(x) = \log_2 x$	$g(x) = -2\log_2(x-1) - 2$
Asymptote	$x = 0$	$x = 1$
Reference point	$(1, 0)$	$(1+1, -2(0)-2) = (2, -2)$
Reference point	$(2, 1)$	$(2+1, -2(1)-2) = (3, -4)$

Logarithmic Functions

Identify the transformations of the graph of $f(x) = \log_b x$ that produce the graph of the given function $g(x)$.

$$g(x) = \frac{1}{2}\log_2(x+1) + 2$$

The transformations of the graph of $f(x) = \log_2 x$ that produce the graph of $g(x)$ are

- **a vertical compression by a factor of $\frac{1}{2}$**
- **a translation of 1 unit to the left and 2 units up**

Function	$f(x) = \log_2 x$	$g(x) = \frac{1}{2}\log_2(x+1) + 2$
Asymptote	$x = 0$	$x = -1$
Reference point	$(1, 0)$	$\left(1 - 1, \frac{1}{2}(0) + 2\right) = (0, 2)$
Reference point	$(2, 1)$	$\left(2 - 1, \frac{1}{2}(1) + 2\right) = \left(1, 2\frac{1}{2}\right)$

Logarithmic Functions

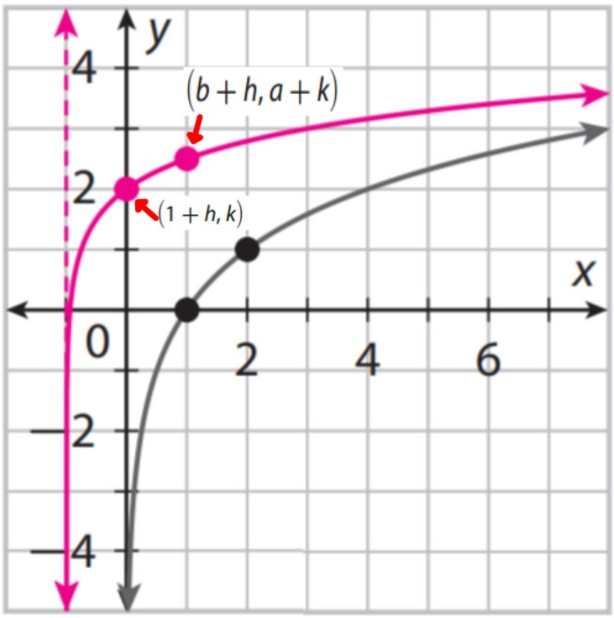
Graphing and Analyzing $f(x) = g(x) = \frac{1}{2}\log_2(x+1) + 2$

First reference point

$\left(1 - 1, \frac{1}{2}(0) + 2\right) = (0, 2)$

Second reference point

$\left(2 - 1, \frac{1}{2}(1) + 2\right) = \left(1, 2\frac{1}{2}\right)$



Domain: $\{x \mid x > -1\}$

Range: $\{y \mid -\infty < y < +\infty\}$

Logarithmic Functions

Graphing and Analyzing $g(x) = -2\log_2(x-1) - 2$

First reference point

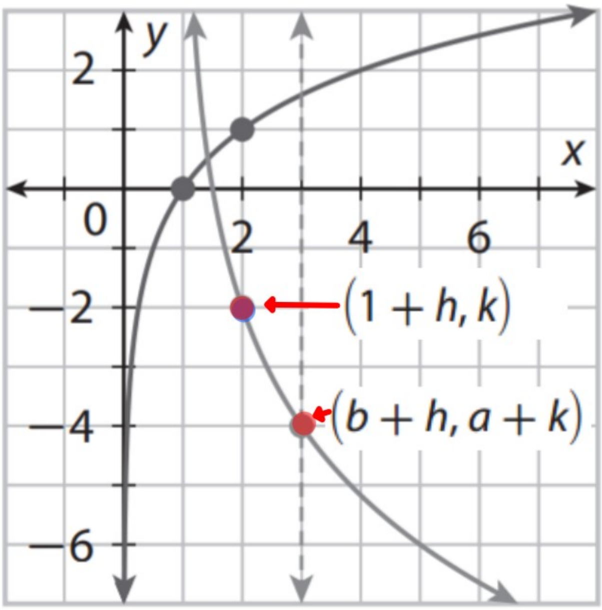
$(1+1, -2(0)-2) = (2, -2)$

Second reference point

$(2+1, -2(1)-2) = (3, -4)$

Domain: $\{x \mid x > 1\}$

Range: $\{y \mid \infty < y < +\infty\}$

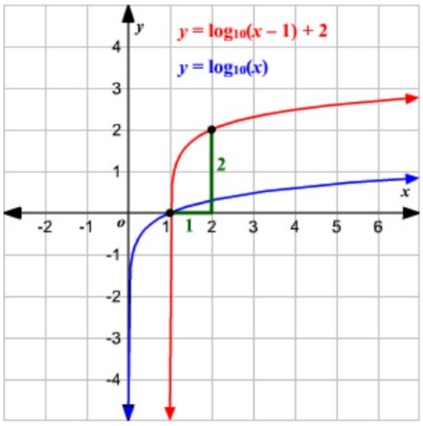


Logarithmic Functions

Any Questions?

Logarithmic Functions

Objectives To graph logarithmic functions



Classwork

Worksheet 15.2

A **logarithmic function** is the inverse of an exponential function. The graph shows $y = 10^x$ and its inverse $y = \log x$. Note that (0, 1) and (1, 10) are on the graph of $y = 10^x$, and that (1, 0) and (10, 1) are on the graph of $y = \log x$.